21. Fes servir la definició de logaritme i calcula:

$$\log_a P = x \qquad a^x = P$$

a) $\log_2 8 = x \qquad 2^x = 8 \quad ; \quad 2^x = 2^3 \quad ; \quad x = 3$

b) $\log_2 1024 = x \qquad 2^x = 1024 \quad ; \quad 2^x = 2^{10} \quad ; \quad x = 10$

c) $\log_3 243 = x \qquad 3^x = 243 \quad ; \qquad 3^x = 3^5 \quad ; \quad x = 5$

d) $\log_5 125 = x \qquad 5^x = 125 \quad ; \qquad 5^x = 5^3 \quad ; \quad x = 3$

e) $\log_5 \dfrac{1}{625} = x \qquad 5^x = \dfrac{1}{5^4} \qquad 5^x = 5^{-4} \quad x = -4$

f) $\log 10000 = x \qquad 10^x = 10^4 \qquad x = 4$

g) $\log_{4/5} 4/5 = x \qquad \left(\dfrac{4}{5}\right)^x = \dfrac{4}{5} \qquad x = 1$

h) $\log 1000 = x \qquad 10^x = 10^3 \qquad x = 3$

i) $\log_3 \dfrac{1}{9} = x \qquad 3^x = \dfrac{1}{3^2} \qquad 3^x = 3^{-2} \qquad x = -2$

j) $\log_3 81 = x \qquad 3^x = 81 \quad ; \quad 3^x = 3^4 \quad ; \quad x = 4$

k) $\log_2 \dfrac{1}{16} = x \qquad 2^x = \dfrac{1}{16} \qquad 2^x = \dfrac{1}{2^4} \qquad 2^x = 2^{-4} \qquad x = -4$

l) $\log_4 64 = x \qquad 4^x = 4^3 \qquad x = 3$

m) $\log_{1/4} 64 = x \qquad \left(\dfrac{1}{4}\right)^x = 64 \qquad \left(\dfrac{1}{4}\right)^x = 4^3 \qquad 4^{-x} = 4^3 \qquad x = -3$

n) $\log_2 32 = x \qquad 2^x = 2^5 \qquad x = 5$

p) $\log_{\frac{2}{3}} \dfrac{4}{9} = x \qquad \left(\dfrac{2}{3}\right)^x = \dfrac{4}{9} \qquad \left(\dfrac{2}{3}\right)^x = \left(\dfrac{2}{3}\right)^2 \qquad x = 2$

o) $\log_{2/3} \dfrac{27}{8} = x \qquad \left(\dfrac{2}{3}\right)^x = \dfrac{27}{8} \qquad \left(\dfrac{2}{3}\right)^x = \left(\dfrac{3}{2}\right)^3 \qquad \left(\dfrac{3}{2}\right)^{-x} = \left(\dfrac{3}{2}\right)^3 \qquad x = -3$

q) $\log 0,0001 = x \qquad 10^x = 10^{-4} \qquad x = -4$

r) $\log_{1/7} \dfrac{1}{49} = x \qquad \left(\dfrac{1}{7}\right)^x = \dfrac{1}{49} \qquad \left(\dfrac{1}{7}\right)^x = \left(\dfrac{1}{7}\right)^2 \qquad x = 2$

s) $\log_{325} 1 = x$ $\qquad$ $325^x = 1$ $\qquad$ $x = 0$

t) $\ln e^3 = x$ $\qquad$ $e^x = e^3$ $\qquad$ $x = 3$

u) $\ln \dfrac{1}{e^7} = x$ $\qquad$ $e^x = \dfrac{1}{e^7}$ $\qquad$ $e^x = e^{-7}$ $\quad$ $x = -7$

v) $\log_{1/2} 32 = x$ $\qquad$ $\left(\dfrac{1}{2}\right)^x = 32$ $\quad$ $2^{-x} = 2^5$ $\quad$ $x = -5$

w) $\log_5 \sqrt{125} = x$ $\qquad$ $5^x = \sqrt{125}$; $\quad$ $5^x = \sqrt{5^3}$; $\quad$ $5^x = 5^{3/2}$; $\qquad$ $x = \dfrac{3}{2}$

2.2 Trobeu el valor de 'x'

a) $\log_x 81 = 4$ $\qquad$ $x^4 = 81$; $\quad$ $x^4 = 3^4$; $\quad$ $x = 3$

b) $\log_x \dfrac{1}{e} = -1$ $\qquad$ $x^{-1} = \dfrac{1}{e}$; $\quad$ $\dfrac{1}{x} = \dfrac{1}{e}$; $\quad$ $x = e$

c) $\log_2 x = 5$ $\qquad$ $2^5 = x$; $\quad$ $x = 32$

d) $\ln x = -2$ $\qquad$; $\quad$ $e^{-2} = x$; $\qquad$ $x = e^{-2} = \dfrac{1}{e^2}$

e) $\log_7 \dfrac{1}{49} = x$; $\quad$ $7^x = \dfrac{1}{49}$; $\quad$ $7^x = 7^{-2}$; $\quad$ $x = -2$

f) $\log_x 4 = \dfrac{1}{16}$; $\quad$ $x^{1/16} = 4$; $\quad$ $\sqrt[16]{x} = 4$; $\quad$ $x = 4^{16} = 4294967296$

g) $\log_3 27 = x$; $\quad$ $3^x = 27$; $\quad$ $3^x = 3^3$; $\quad$ $x = 3$

h) $\log_x \dfrac{9}{25} = 2$; $\quad$ $x^2 = \dfrac{9}{25}$; $\quad$ $x^2 = \left(\dfrac{3}{5}\right)^2$; $\quad$ $x = \dfrac{3}{5}$

i) $\log 100 = x$; $\quad$ $10^x = 100$; $\quad$ $10^x = 10^2$; $\quad$ $x = 2$

j) $\log x = 4$; $\quad$ $10^4 = x$; $\quad$ $x = 10000$

k) $\ln e^2 = x$; $e^x = e^2$; $x = 2$

l) $\ln 1 = x$; $e^x = 1$; $x = 0$

m) $\log_x 3 = 1$ $x^1 = 3$; $x = 3$

n) $\log_x \frac{3}{5} = -1$; $x^{-1} = \frac{3}{5}$; $x^{-1} = \left(\frac{5}{3}\right)^{-1}$; $x = \frac{5}{3}$

3. Quines relacions són certes?

a) $\log(a+b) = \log a + \log b$ NO $(\log a + \log b = \log a \cdot b)$

b) $\log \frac{a}{b} = \log a - \log b$ SÍ

c) $\log a / \log b = \log(a-b)$ NO

d) $\log a^b = b \log a$ SÍ

e) $\log(ab)^c = c \log a + c \log b$ SÍ

4. Expressa, aplicant les propietats dels logaritmes, els següents logaritmes en funció de $\log 2$ i $\log 3$.

a) $\log 60 = \log 2^2 \cdot 3 \cdot \frac{10}{2} = 2\log 2 + \log 3 + \log 10 - \log 2 = \log 2 + \log 3 + 1$

b) $\log 75 = \log 5^2 \cdot 3 = \log \left(\frac{10}{2}\right)^2 \cdot 3 = 2\log 10 - 2\log 2 + \log 3 = 2 - 2\log 2 + \log 3$

c) $\log 900 = \log 3^2 \cdot 100 = 2\log 3 + 2$

d) $\log 18 = \log 2 \cdot 3^2 = \log 2 + 2\log 3$

e) $\log 40 = \log 2^2 \cdot 10 = 2\log 2 + \log 10 = 2\log 2 + 1$

f) $\log 270 = \log 3^3 \cdot 10 = 3\log 3 + 1$

g) $\log 150 = \log 3 \cdot \frac{10}{2} \cdot 10 = \log 3 - \log 2 + 2$

h) $\log 8 = \log 2^3 = 3\log 2$

5. Expresseu els següents logaritmes en funció de $\log 9$ i $\log 8$

a) $\log 72 = \log 8 \cdot 9 = \log 8 + \log 9$

b) $\log 900 = \log 9 \cdot 100 = \log 9 + \log 100 = \log 9 + 2$

c) $\log 64 = \log 8^2 = 2 \log 8$

d) $\log 45/4 = \log \dfrac{90}{8} = \log \dfrac{9 \cdot 10}{8} = \log 9 + 1 - \log 8$

e) $\log_9 600 = \dfrac{\log 600}{\log 9} = \dfrac{\log 8 \cdot 3 \cdot 5^2}{\log 9}$ ✓

f) $\log_8 800 = \dfrac{\log 8 \cdot 100}{\log 8} = \dfrac{\log 8 + 2}{\log 8}$

6. Expresseu en logaritme decimal:

a) $\log_2 3 = \dfrac{\log 3}{\log 2}$

b) $\log_2 270 = \dfrac{\log 270}{\log 2}$

c) $\log_3 2 = \dfrac{\log 2}{\log 3}$

d) $\log_3 810 = \dfrac{\log 810}{\log 3}$

e) $\log_{27} 9 = \dfrac{\log 9}{\log 27}$

f) $\log_5 3 = \dfrac{\log 3}{\log 5}$

g) $\log_{18} 216 = \dfrac{\log 216}{\log 18}$

h) $\ln 5 = \dfrac{\log 5}{\log e}$

7. Reduïu a un únic logaritme:

a) $\log 3a - 7 \log a + 2 \log 10 = \log \dfrac{3a \cdot 10^2}{a^7} = \log \dfrac{3 \cdot 10^2}{a^6}$

b) $\log 16 - \log a + 32 \log 2 = \log \dfrac{16 \cdot 2^{32}}{a} = \log \dfrac{2^{36}}{a}$

c) $4(\log 3 + \log a - 2 \log b) = \log \left(\dfrac{3a}{b^2}\right)^4 = \log \dfrac{3^4 a^4}{b^8}$

d) $\log x + 9(\log y + \log x) = \log x \cdot y^9 + x^9 = \log y^9 \cdot x^{10}$