

21. Fes servir la definició de logaritme i calcula:

$$\log_a P = x \quad a^x = P$$

a) $\log_2 8 = x \quad 2^x = 8 \quad x = 3$

b) $\log_2 1024 = x \quad 2^x = 1024 \quad x = 10$

c) $\log_3 243 = x \quad 3^x = 243 \quad x = 5$

d) $\log_5 125 = x \quad 5^x = 125 \quad x = 3$

e) $\log_5 \frac{1}{625} = x \quad 5^x = \frac{1}{5^4} \quad 5^x = 5^{-4} \quad x = -4$

f) $\log 10000 = x \quad 10^x = 10^4 \quad x = 4$

g) $\log_{4/5} 4/5 = x \quad \left(\frac{4}{5}\right)^x = \frac{4}{5} \quad x = 1$

h) $\log 1000 = x \quad 10^x = 10^3 \quad x = 3$

i) $\log_3 \frac{1}{9} = x \quad 3^x = \frac{1}{3^2} \quad 3^x = 3^{-2} \quad x = -2$

j) $\log_3 81 = x \quad 3^x = 81 \quad x = 4$

k) $\log_2 \frac{1}{16} = x \quad 2^x = \frac{1}{16} \quad 2^x = \frac{1}{2^4} \quad 2^x = 2^{-4} \quad x = -4$

l) $\log_4 64 = x \quad 4^x = 4^3 \quad x = 3$

m) $\log_{1/4} 64 = x \quad \left(\frac{1}{4}\right)^x = 64 \quad \left(\frac{1}{4}\right)^x = 4^3 \quad 4^{-x} = 4^3 \quad x = -3$

n) $\log_2 32 = x \quad 2^x = 2^5 \quad x = 5$

p) $\log_{\frac{2}{3}} \frac{4}{9} = x \quad \left(\frac{2}{3}\right)^x = \frac{4}{9} \quad \left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^2 \quad x = 2$

o) $\log_{2/3} \frac{27}{8} = x \quad \left(\frac{2}{3}\right)^x = \frac{27}{8} \quad \left(\frac{2}{3}\right)^x = \left(\frac{3}{2}\right)^3 \quad \left(\frac{3}{2}\right)^{-x} = \left(\frac{3}{2}\right)^3 \quad x = -3$

q) $\log 0.0001 = x \quad 10^x = 10^{-4} \quad x = -4$

r) $\log_{1/7} \frac{1}{49} = x \quad \left(\frac{1}{7}\right)^x = \frac{1}{49} \quad \left(\frac{1}{7}\right)^x = \left(\frac{1}{7}\right)^2 \quad x = 2$

$$s) \log_{325} 1 = x$$

$$325^x = 1 \quad x=0$$

$$t) \ln e^3 = x$$

$$e^x = e^3 \quad x=3$$

$$u) \ln \frac{1}{e^7} = x$$

$$e^x = \frac{1}{e^7} \quad e^x = e^{-7} \quad x=-7$$

$$v) \log_{\frac{1}{2}} 32 = x$$

$$\left(\frac{1}{2}\right)^x = 32 \quad 2^{-x} = 2^5 \quad x=-5$$

$$w) \log \sqrt{125} = x$$

$$10^x = \sqrt{125}; \quad 10^x = \sqrt{5^3} \quad 10^x = 5^{3/2} \quad 10^x = / \quad \text{No amb la definició de logaritme}$$

22. Trobeu el valor de 'x'

$$a) \log_x 81 = 4$$

$$x^4 = 81; \quad x^4 = 3^4; \quad x=3$$

$$b) \log_x \frac{1}{e} = -1$$

$$x^{-1} = \frac{1}{e}; \quad \frac{1}{x} = \frac{1}{e}; \quad x=e$$

$$c) \log_2 x = 5$$

$$2^5 = x; \quad x=32$$

$$d) \log_x 4 = \frac{1}{16}$$

$$x^{1/16} = 4; \quad \sqrt[16]{x} = 4; \quad x = 4^{16} = 4294967296$$

$$e) \log_3 27 = x$$

$$3^x = 27; \quad 3^x = 3^3; \quad x=3$$

$$f) \log_x \frac{9}{25} = 2$$

$$x^2 = \frac{9}{25}; \quad x^2 = \frac{3^2}{5^2} = \left(\frac{3}{5}\right)^2; \quad x = \frac{3}{5}$$

$$g) \log 100 = x$$

$$10^x = 100; \quad 10^x = 10^2; \quad x=2$$

$$h) \log x = 4$$

$$10^4 = x; \quad x=10000$$

$$i) \log_7 \frac{1}{49} = x$$

$$7^x = \frac{1}{49}; \quad \left(\frac{1}{7}\right)^{-x} = \left(\frac{1}{7}\right)^2; \quad x=-2$$

$$j) \log_x 3 = 1$$

$$x^1 = 3; \quad x=3$$

$$k) \log_x \frac{3}{5} = -1 \quad x^{-1} = \frac{3}{5} \quad ; \quad x = \frac{5}{3}$$

$$l) \ln e^2 = x \quad e^x = e^2 \quad ; \quad x = 2$$

$$m) \ln 1 = x \quad e^x = 1 \quad ; \quad x = 0$$

$$n) \ln x = -2 \quad e^{-2} = x \quad ; \quad x = \frac{1}{e^2}$$

23. Quines relacions són certes?

$$a) \log(a+b) = \log a + \log b \quad \text{NO} \quad (\log a + \log b = \log a \cdot b)$$

$$b) \log \frac{a}{b} = \log a - \log b \quad \text{SÍ}$$

$$c) \log a / \log b = \log(a-b) \quad \text{NO}$$

$$d) \log a^b = b \log a \quad \text{SÍ}$$

$$e) \log(ab)^c = c \log a + c \log b \quad \text{SÍ}$$

24. Expressa, aplicant les propietats dels logaritmes, els següents logaritmes en funció de $\log 2$ i $\log 3$.

$$a) \log 60 = \log 2^2 \cdot 3 \cdot \frac{10}{2} = 2 \log 2 + \log 3 + \log 10 - \log 2 = \log 2 + \log 3 + 1$$

$$b) \log 75 = \log 5^2 \cdot 3 = \log \left(\frac{10}{2}\right)^2 \cdot 3 = 2 \log 10 - 2 \log 2 + \log 3 = 2 - 2 \log 2 + \log 3$$

$$c) \log 900 = \log 3^2 \cdot 100 = 2 \log 3 + 2$$

$$d) \log 18 = \log 2 \cdot 3^2 = \log 2 + 2 \log 3$$

$$e) \log 40 = \log 2^2 \cdot 10 = 2 \log 2 + \log 10 = 2 \log 2 + 1$$

$$f) \log 270 = \log 3^3 \cdot 10 = 3 \log 3 + 1$$

$$g) \log 150 = \log 3 \cdot \frac{10}{2} \cdot 10 = \log 3 - \log 2 + 2$$

$$h) \log 8 = \log 2^3 = 3 \log 2$$

25. Expressen els següents logaritmes en funció de $\log 9$ i $\log 8$

$$a) \log 72 = \log 8 \cdot 9 = \log 8 + \log 9$$

$$b) \log 900 = \log 9 \cdot 100 = \log 9 + \log 100 = \log 9 + 2$$

$$c) \log 64 = \log 8^2 = 2 \log 8$$

$$d) \log 45/4 = \log \frac{90}{8} = \log \frac{9 \cdot 10}{8} = \log 9 + 1 - \log 8$$

$$e) \log_9 600 = \frac{\log 600}{\log 9} = \frac{\log 8 \cdot 3 \cdot 5^2}{\log 9}$$

$$f) \log_8 800 = \frac{\log 8 \cdot 100}{\log 8} = \frac{\log 8 + 2}{\log 8}$$

26. Expressen en logaritme decimal:

$$a) \log_2 3 = \frac{\log 3}{\log 2}$$

$$e) \log_{27} 9 = \frac{\log 9}{\log 27}$$

$$b) \log_2 270 = \frac{\log 270}{\log 2}$$

$$f) \log_5 3 = \frac{\log 3}{\log 5}$$

$$c) \log_3 2 = \frac{\log 2}{\log 3}$$

$$g) \log_{10} 216 = \frac{\log 216}{\log 18}$$

$$d) \log_3 810 = \frac{\log 810}{\log 3}$$

$$h) \ln 5 = \frac{\log 5}{\log e}$$

27. Reduïu a un únic logaritme:

$$a) \log 3a - 7 \log a + 2 \log 10 = \log \frac{3a \cdot 10^2}{a^7} = \log \frac{3 \cdot 10^2}{a^6}$$

$$b) \log 16 - \log a + 32 \log 2 = \log \frac{16 \cdot 2^{32}}{a} = \log \frac{2^{36}}{a}$$

$$c) 4(\log 3 + \log a - 2 \log b) = \log \left(\frac{3a}{b^2} \right)^4 = \log \frac{3^4 a^4}{b^8}$$

$$d) \log x + 9(\log y + \log x) = \log x \cdot y^9 + x^9 = \log y^9 \cdot x^{10}$$